II. Steady and irrotational flow:- If the motion is steady then $\frac{\partial \phi}{\partial t} = 0$

$$\Rightarrow \int \frac{dP}{\rho} + \frac{1}{2} q^2 + \Omega = \text{constant}$$

If the fluid is homogeneous and incompressible then variation of density remain constant. then

$$\boxed{\frac{P}{\rho} + \frac{1}{2} q^2 + \Omega = \text{constant}}$$

This is known as Bernoulli's equation for steady and irrotational flow.

—x—

Euler's equations of motion:-

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X - \frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = Y - \frac{1}{\rho}\frac{\partial P}{\partial y}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = Z - \frac{\partial P}{\partial z}$$

—x—

Similarly about y axis and z axis we have

$$-\frac{\partial}{\partial y}(\rho v)\,\delta x\,\delta y\,\delta z \quad \text{along } y \text{ axis}$$

And $-\frac{\partial}{\partial z}(\rho w)\,\delta x\,\delta y\,\delta z$ along z axis

Total excess of mass through th $\square^{d}$

$$= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]\delta x\,\delta y\,\delta z \quad \text{— (1)}$$

Total mass inside the Parallelopiped

$$= \rho\,\delta x\,\delta y\,\delta z$$

Rate of increase in the mass inside the Parallopiped

$$= \frac{\partial}{\partial t}(\rho\,\delta x\,\delta y\,\delta z) \Rightarrow \frac{\partial \rho}{\partial t}\,\delta x\,\delta y\,\delta z \quad \text{— (ii)}$$

From (1) And (2)

Rate of mass accumulation = Rate of mass in - Rate of mass out

$$\frac{\partial \rho}{\partial t}\,\delta x\,\delta y\,\delta z = -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]\delta x\,\delta y\,\delta z$$

or $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\delta x\,\delta y\,\delta z \neq 0$$

or $$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

i.e. $$\frac{D\rho}{Dt} + \rho\;\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

or $$\boxed{\frac{D\rho}{Dt} + \rho(\nabla q) = 0}$$

Knows as eq^{n}. of continuity in cartesian c

—o—

$$= K\left\{\frac{\partial}{\partial x}\left(\frac{Ax}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{Ay}{x^2+y^2}\right)\right\}$$

$$= KA\left\{\frac{(x^2+y^2)-2x^2}{(x^2+y^2)^2} + \frac{(x^2+y^2)-2y^2}{(x^2+y^2)^2}\right\}$$

Hence the motion is of Potential kind $= 0$

$\Rightarrow$ motion is irrotational

(4) $\Rightarrow$ Velocity Potential exists.

$$q = -\nabla\phi$$

$$\Rightarrow \frac{\partial\phi}{\partial x} = -u = \frac{Ay}{x^2+y^2}$$

$$\frac{\partial\phi}{\partial y} = -v = \frac{Ax}{x^2+y^2}$$

$$\frac{\partial\phi}{\partial z} = -w = 0 \Rightarrow \phi \text{ independent of } z$$

$$\boxed{\phi = \phi(x,y)}$$

(5) $\frac{Ay}{x^2+y^2} \Rightarrow \phi = A\tan^{-1}\frac{x}{y} + f(y)$

$\cancel{\frac{Ax}{x^2+y^2} \Rightarrow \phi = \frac{-Ax}{x^2+y^2} = \frac{A}{1+\frac{x^2}{y^2}} (-x-}$

$$\frac{\partial\phi}{\partial y} = f'(y) - \frac{Ax}{x^2+y^2}$$

$$f'(y) = 0$$

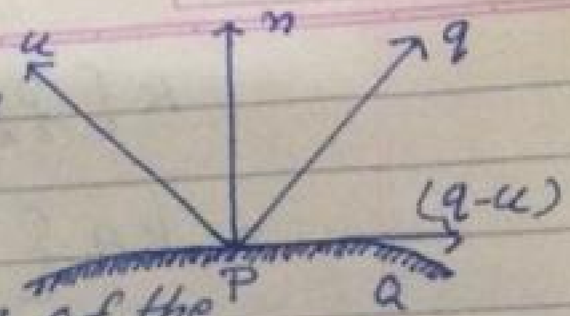
$$f(y) = \text{constant}$$

$$\phi(x,y) = A\tan^{-1}\frac{x}{y}$$

c is an absalutly Constant.

Boundary Surface:-

u = velocity of surface
q = velocity of fluid



Consider the equation of the Boundary Surface at the Point P

$$F(u,t)=0$$

Let Q be the Velocity of surface at Point P

Consider n is the unit normal vector drawn at the Point P on the Boundary surface

Since there must be no relative normal velocity at P between the Boundary and the fluid so we must have

$$q\cdot n = u\cdot n$$

$$\Rightarrow (q-u)\cdot n = 0$$

$$\Rightarrow (q-u)\nabla F \qquad (n=\nabla F)$$

At The Point Q

$$F(r+\delta r,\ t+\delta t) = 0$$

Expanding by Taylor's theorem

$$F(r,t) + \delta r\nabla F + \delta t\cdot\frac{\partial F}{\partial t} = 0$$

$$\delta r\nabla F + \delta r\cdot\frac{\partial F}{\partial t} = 0$$

$$\Rightarrow \frac{\partial F}{\partial t} + \frac{\partial r}{\partial t}\nabla F = 0$$

$$\Rightarrow \frac{\partial F}{\partial t} + u\nabla F = 0$$

As $\partial t \to 0,\ \partial r \to 0$

$$u = \frac{\partial r}{\partial t}$$

$$\Rightarrow \boxed{\frac{\partial F}{\partial t} + q\nabla F = 0}$$ Known as eq^n. of Boundary surface

I.m.P.
Example 8 26 obtain Path lines and streak lines.

$$q = \left(\frac{x}{t}, y, 0\right)$$

$$q = iu + jv + kw$$
$$= \left(\frac{x}{t}, y, 0\right)$$

$$u = \frac{dx}{dt} = \frac{x}{t} \Rightarrow \frac{dx}{x} = \frac{dt}{t}$$

Integrating we have

$$\log x = \log t + \log A$$
$$x = At$$

$$v = \frac{dy}{dt} = y \Rightarrow \frac{dy}{y} = dt$$

Integrating we have

$$\log y = t + \log B$$
$$y = Be^{t}$$

$W = \frac{dz}{dt} = 0 \Rightarrow z = c$ constant is Independent of time

Eq^n of the Path can be determine y Eliminating t between $x, y,$ and z

$$x = At$$
$$y = Be^{t}$$
$$z = c$$

Consider (x_0, y_0, z_0) be the cardinate of the fluid Particle then

$$x_0 = At_0, \quad y_0 = Be^{t_0}, \quad z_0 = c$$
$$\Rightarrow A = \frac{x_0}{t_0}, \quad B = y_0 e^{-t_0}, \quad c = z_0$$